FILTRATION OF A GASIFIED LIQUID

V. Sh. Shagapov UDC 532.08

When a liquid saturated with gas (n-hexane with carbon dioxide) is filtered through a porous medium, a sharp increase in the flow rate (by ~2-3 times) has been observed [1] when the gas pressure at the exit of the porous medium is reduced close to that to the gas evolution pressure; when the exit pressure is further reduced, the flow rate drops. Here, we present a "gas bearing" model to explain this effect (the sharp increase in the liquid flow rate). Expressions are obtained for the relative phase permeability within the framework of this mechanism, and these are used to construct stationary and self-similar solutions. These solutions are compared with experimental data and analyzed.

1. Basic Equations. We now examine the liquid flow in a porous medium in the presence of gas evolution. The subscripts 1 and 2 denote parameters for the liquid and gas phases. The equations for the conservation of mass for two-phase filtration have the form [2-4]

$$\frac{\partial}{\partial t}(\rho_1^0 m S_1) + \nabla\cdot(\rho_1^0 m S_1 \mathbf{v}_1) = -I, \quad \frac{\partial}{\partial t}(\rho_2^0 m S_2) + \nabla\cdot(\rho_2^0 m S_2 \mathbf{v}_2) = I, \tag{1.1}$$

where ρ_i^0, S_i, $\mathbf{v}_i$, m, and I are the density, saturation, velocity, porosity, and the gas evolution rate for a unit volume of porous medium. If we assume that only dissolved gas is considered in the mass transfer between the phases, then the continuity equation for the dissolved gas is written as

$$\frac{\partial}{\partial t}(\rho_1^0 g m S_1) + \nabla\cdot(\rho_1^0 g m S_1 \mathbf{v}_1) = -I, \tag{1.2}$$

where g is the mass concentration of the dissolved gas. The momentum equations for the gas are a generalization of Darcy's law:

$$mS_i\mathbf{v}_i = -\frac{kK_i}{\mu_i}\nabla p \qquad (i = 1, 2). \tag{1.3}$$

Here k, K_i, and μ_i are the absolute permeability of the porous medium, the relative permeability, and the dynamic viscosity.

We use Henry's law for the concentration of the dissolved gas versus the pressure in the gas-evolution region and assume that the gas phase satisfies the Clapeyron–Mendeleev equation:

$$g = Gp, \qquad p = \rho_2^0 RT. \tag{1.4}$$

We also neglect the dependence of the parameters G, R, m, and μ_i on pressure and assume that the process is isothermal ($T = T_0$ = const).

Hereafter we exclude I from Eqs. (1.1) and (1.2) and, instead of three equations, use two:

$$\frac{\partial}{\partial t}[m(\rho_1^0 S_1 + \rho_2^0 S_2)] + \nabla\cdot[m(\rho_1^0 S_1\mathbf{v}_1 + \rho_2^0 S_2 \mathbf{v}_2)] = 0; \tag{1.5}$$

$$\frac{\partial}{\partial t}[m\rho_1^0(1-g)S_1] + \nabla\cdot[m\rho_1^0(1-g)S_1\mathbf{v}_1] = 0. \tag{1.6}$$

2. Relative Phase Permeability. Equations (1.3)-(1.6) form a closed system when the relative phase permeabilities K_i are given as functions of S_i. In order to obtain these equations, we use a "gas bearing" model, according to which we assume that the gas phase in the porous medium basically flows in a layer on the pore wall in the zone where gas evolution starts. According to this hypothesis, the best conditions for evolution of the gas dissolved in the liquid occur in the contact zone between the liquid and the wall of the porous channels (for example due to sites for nucleus formation).

Tyumen'. Translated from Prikladnaya Mekhanika i Tekhnicheskaya Fizika, No. 5, pp. 97-106, September-October, 1993. Original article submitted December 17, 1991; revision submitted May 21, 1992.

 0021-8944/93/3405-0686$12.50 © 1994 Plenum Publishing Corporation

We now use this "gas bearing" model to examine an auxiliary problem of annular layered gas–liquid flow medium in the inertialess approximation in a cylindrical channel. We assume that the gas phase flows in an annular layer on the channel wall. Moreover, we assume that the velocity distribution follows Poisseuille flow in every section of the channel and satisfies the equation

$$\frac{\mu_i}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_i'}{\partial r}\right)=\frac{dp}{dx}\qquad \left(i=\begin{cases}1, & 0<r<a-\delta\\ 2, & a-\delta<r<a\end{cases}\right), \tag{2.1}$$

where v_i' is the radial velocity distribution for the cross section at x, and a and δ are the radius of the channel and the thickness of the annular gas layer. By assuming that (the gas) sticks to the channel wall, equality of the velocities and the tangential stresses on the interphase surface

$$v_2'=0\quad (r=a),\quad v_1'=v_2',\quad \mu_1\frac{\partial v_1'}{\partial r}=\mu_2\frac{\partial v_2'}{\partial r}\quad (r=a-\delta), \tag{2.2}$$

gives a velocity distribution

$$v_1'=\left[\frac{r^2-(a-\delta)^2}{4\mu_1}+\frac{(a-\delta)^2-a^2}{4\mu_2}\right]\frac{dp}{dx},\quad v_2'=\frac{r^2-a^2}{4\mu_2}\frac{dp}{dx}. \tag{2.3}$$

We introduce the flow-rate-averaged velocities:

$$\begin{gathered}v_i=q_i/F_i,\quad q_i=\int_{F_i} v_i'dF_i'\quad (i=1,\,2),\\ F_1=\pi\,(a-\delta)^2,\quad F_2=\pi\,[a^2-(a-\delta)^2],\quad F=F_1+F_2,\end{gathered} \tag{2.4}$$

where F_i is the cross-sectional area of the channel relative to the i-th phase. By substituting (2.3) into (2.4), we obtain a relationship between the flow-rate-averaged velocities and the pressure gradient:

$$v_1=-\frac{\tilde\mu F_1+2F_2}{8\pi\tilde\mu\mu_1}\frac{dp}{dx},\quad v_2=-\frac{F_2}{8\pi\mu_2}\frac{dp}{dx}\quad \left(\tilde\mu=\frac{\mu_2}{\mu_1}\right). \tag{2.5}$$

We compare Eqs. (2.5) and (1.3). The relative phase permeabilities are taken to be those values which provide the same flow velocities in both the cylindrical channel and the porous medium for the same pressure gradients. In the comparison, it is natural to assume that the ratio k/m in Darcy's law (1.3) corresponds to $F/8\pi$ in (2.5) and that the phase saturation corresponds to F_i/F. Then the phase permeabilities can be written as

$$K_1=(1-S_2)\,[\tilde\mu+(2-\tilde\mu)\,S_2]/\tilde\mu,\quad K_2=S_2^2. \tag{2.6}$$

It follows from these expressions that $K_1 > 1$ for $S_2 > 0$. This situation is not very usual from the viewpoint of filtering gas–liquid media, where each phase completely occupies the pore volume it flows in. We note that the layered flow model used in obtaining (2.6) is not realized in a "pure" sense. This hypothesis is only a limiting idealization of the gas–liquid flow when the dominant part of the gas phase separates and flows in a layer on the wall. Furthermore, we note that the flow structure in the porous medium is not affected by gravity in these filtration processes, because the Bond number $\mathrm{Bo} = \rho_1^0 g d^2/\sigma \ll 1$ (here g is the acceleration due to gravity, d is the characteristic pore diameter, and σ is the surface tension). Therefore the capillary forces dominate the force of gravity.

If, we invert the previous case and assume that the liquid flows next to the wall and the gas in the middle, we have for the relative phase permeabilities

$$K_1=S_1^2,\quad K_2=(1-S_1)\,[1+(2\tilde\mu-1)\,S_1]. \tag{2.7}$$

For an equal-velocity flow model ($v_1 = v_2$), we have

$$K_1=S_1/\tilde\mu_*,\quad K_2=(1-S_1)\,\tilde\mu/\tilde\mu_*\quad (\tilde\mu_*=\mu_*/\mu_1), \tag{2.8}$$

where μ_* is the effective viscosity for the gasified liquid.

3. Steady-State Filtration. We now examine one-dimensional steady-state filtration flow. It follows from (1.5) and (1.6) that

$$\frac{d}{dx}\,[x^{\nu-1}\,(\rho_1^0S_1v_1+\rho_2^0S_2v_2)]=0,\quad \frac{d}{dx}\,[x^{\nu-1}\rho_1^0\,(1-g)\,S_1v_1]=0. \tag{3.1}$$

The values $\nu = 1$, 2, and 3 correspond to the plane, cylindrical, and spherical cases. By integrating these equations we find

$$x^{\nu-1}(\rho_1^0 S_1 v_1 + \rho_2^0 S_2 v_2) = x_s^{\nu-1}\rho_1^0 v_{1s}, \quad x^{\nu-1}\rho_1^0(1-g)S_1 v_1 = x_s^{\nu-1}\rho_1^0(1-g_s)v_{1s}, \tag{3.2}$$

where x_s is the coordinate where gas evolution starts. The additional subscript s indicates that the parameter value corresponds to this section ($x = x_s$). The right sides of (3.2) are written for $S_{1s} = 1$ and $S_{2s} = 0$).

From (3.2) and (1.3) we obtain

$$\frac{kK_1}{m\mu_1}\left(\frac{1-g}{1-g_s}\right) = \left(\frac{x_s}{x}\right)^{\nu-1} v_{1s}, \qquad K_2 = \bar{\mu}\,\frac{(g_s-g)\rho_1^0}{(1-g_s)\rho_2^0}K_2. \tag{3.3}$$

The additional parameter G and the quantity RT in (1.4) are written in terms of the parameter values at the section where gas evolution starts:

$$G = g_s/p_s, \qquad RT = p_s/\rho_{2s}^0. \tag{3.4}$$

If Eqs. (2.6), which correspond to the "gas bearing" model, are used for the K_i, then it follows from (3.3), (1.4), and (3.4), that

$$\frac{k}{m\mu_1}\frac{dp}{dx} = -f(p)\,v_{1s}\left(\frac{x_s}{x}\right)^{\nu-1},$$
$$f(p) = \frac{(1-g_s)p_s\left[\sqrt{p_s-(1-\chi\bar{\mu})p} - (1-\bar{\mu})\sqrt{p_s-p}\right]^2}{\chi\bar{\mu}p(p_s-g_s p)}, \qquad \chi = \frac{\rho_{2s}^0(1-g_s)}{\rho_1^0 g_s}, \tag{3.5}$$

from which we find the pressure as a function of the coordinate

$$\int_{p_s}^{p} f^{-1}(p)\,dp = -\frac{m\mu_1 v_{1s}}{k}\int_{x_s}^{x}(x_s/x)^{\nu-1}\,dx. \tag{3.6}$$

Here the velocities and the gas saturations are given as functions of the pressure and the coordinates by

$$v_1 = \frac{(1-g_s)v_{1s}}{(1-g_s p/p_s)(1-S_2)}\left(\frac{x_s}{x}\right)^{\nu-1}, \qquad v_2 = \frac{S_2}{\bar{\mu}+(2-\bar{\mu})S_2}v_1,$$
$$S_2 = \frac{\bar{\mu}\sqrt{p_s-p}}{\sqrt{p_s-(1-\chi\bar{\mu})p}-(1-\bar{\mu})\sqrt{p_s-p}}, \qquad S_1 = 1 - S_2. \tag{3.7}$$

Let p_0 be the pressure at $x = x_0$ and let $p_0 > p_s$. Then the pressure p in the region $x_s < x \leqslant x_0$ is higher than the pressure p_s for the start of gas evolution ($p > p_s$); therefore in this region we have single-phase liquid filtration. If we set $f(p) = 1$ based on (3.5), we obtain

$$p = p_0 - \frac{m\mu_1 v_{1s}}{k}\int_{x_0}^{x}(x_s/x)^{\nu-1}dx \qquad (x_s < x < x_0). \tag{3.8}$$

It follows from Eqs. (3.7) and (3.8) that

$$\left(\int_{x_s}^{x_e} x^{\nu-1}dx\right)\left(\int_{x_0}^{x_s} x^{\nu-1}dx\right)^{-1} = \frac{1}{p_0-p_s}\int_{p_s}^{p_e} f^{-1}(p)\,dp, \tag{3.9}$$

where p_e is the pressure at the exit from the porous medium ($x = x_e$). This equation makes it possible to obtain a relationship for the position of the coordinate for the start of gas evolution as a function of p_0, p_s, and p_e. The integration constant v_{1s}, which is the liquid-phase velocity in the section where gas evolution starts, can be determined from

$$v_{1s} = \frac{k(p_0-p_s)}{m\mu_1}\left[\int_{x_0}^{x_s}(x_s/x)^{\nu-1}dx\right]^{-1}, \tag{3.10}$$

which follows from (3.8) with $x = x_s$ and $p = p_s$.

If there is no gas evolution, we have the filtration velocity of the liquid through the boundary of the porous medium ($x = x_e$) from Eq. (3.8)

$$u_e^* = mv_e^* = -\frac{k(p_0 - p_e)}{\mu_1}\left[\int_{x_0}^{x_e}\left(\frac{x_s}{x}\right)^{\nu-1} dx\right]^{-1}. \tag{3.11}$$

We now write the expression for the gas and liquid filtration velocities at the boundary of the flow region ($x = x_e$):

$$u_{ie} = mS_{ie}v_{ie} = -\frac{kK_{ie}}{\mu_i}\left(\frac{dp}{dx}\right)_{x_e} \quad (i = 1, 2). \tag{3.12}$$

Considering (2.6) and (3.3), we obtain from (3.12) that

$$u_{1e} = \frac{(1 - g_s)p_e}{p_s - g_s p_e}\left(\frac{x_s}{x_e}\right)^{\nu-1} mv_{1s}, \quad u_{2e} = \frac{p_s - p_e}{\chi p_e} u_{1e}. \tag{3.13}$$

We reduce (3.13) to a dimensionless form by using Eq. (3.10) instead of v_{1s} and by considering (3.11):

$$Q_{1e} = \frac{(1 - g_s)p_s(p_0 - p_s)}{(p_s - g_s p_e)(p_0 - p_e)}\left(\int_{x_0}^{x_e} x^{\nu-1}dx\right)\left(\int_{x_0}^{x_s} x^{\nu-1}dx\right)^{-1},$$

$$Q_{2e} = \frac{p_s - p_e}{\chi p_e} Q_{1e}, \quad Q_{ie} = \frac{u_{ie}}{u_e^*}. \tag{3.14}$$

The parameter $Q_e = (\rho_1^0 u_{1e} + \rho_{2e}^0 u_{2e})/\rho_1^0 u_e^*$, which determines the change in the mass flow rate of the gas–liquid mixture, is obtained by comparison with the case with no gas evolution and with a consideration of (3.14):

$$Q_e = \frac{p_0 - p_s}{p_0 - p_e}\left(\int_{x_0}^{x_e} x^{\nu-1}dx\right)\left(\int_{x_0}^{x_s} x^{\nu-1}dx\right)^{-1}. \tag{3.15}$$

We now consider the two-dimensional plane case ($\nu = 1$) in more detail. We assume that $x_e = 0$ and $x_0 = \ell$. Then the position where gas evolution starts is obtained from (3.9)

$$x_s = l\left(1 - \frac{1}{p_0 - p_s}\int_{p_s}^{p_e} f^{-1}(p)\,dp\right)^{-1}.$$

We write the expressions for the dimensionless flow rates (3.14) and (3.15) in the two-dimensional plane form

$$Q_e = \frac{p_0 - p_s}{p_0 - p_e} - \frac{1}{p_0 - p_e}\int_{p_s}^{p_e} f^{-1}(p)\,dp, \qquad Q_{1e} = \frac{(1 - g_s)p_s}{p_s - g_s p_e} Q_e.$$

The conditions $\tilde{\mu} \ll 1$ and $\chi = O(1)$ are usually fulfilled for most liquids and gases. Then, where the inequality

$$p_s - p \gg \chi\tilde{\mu}p_s,$$

is fulfilled, the expression for the pressure function $f(p)$ from (3.5) can be simplified:

$$f(p) = \frac{\chi\tilde{\mu}(1 - g_s)p_s p}{4(p_s - g_s p)(p_s - p)}. \tag{3.16}$$

By substituting (3.16) into (3.6) we have for $\nu = 1$:

$$\frac{x - x_s}{x_s} = \frac{4}{\chi\tilde{\mu}(1 - g_s)(p_0 - p_s)}\left[(1 + g_s)(p_s - p) + p_s \ln\frac{p}{p_s} + \frac{g_s(p^2 - p_s^2)}{2p_s}\right].$$

Here we have for the flow rate

$$Q_e = \frac{4}{\chi\tilde{\mu}(1 - g_s)(p_0 - p_s)}\left[(1 + g_s)(p_s - p_e) + p_s \ln\frac{p_e}{p_s} + \frac{g_s(p_e^2 - p_s^2)}{2p_s}\right] + \frac{p_0 - p_s}{p_0 - p_e}.$$

We note that the use of (3.16) in Eq. (3.6) corresponds to neglecting the length of the two-phase filtration zone, where p is small compared to p_s.

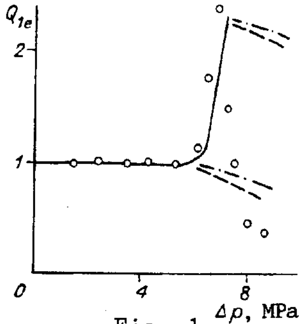

Fig. 1

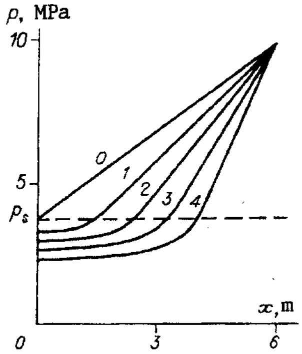

Fig. 2

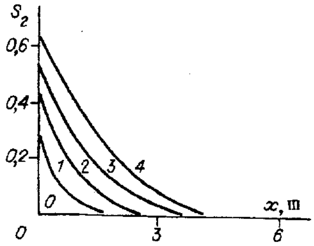

Fig. 3

For generality we present the expression for the pressure function for the inverted flow, where the liquid flows next to the wall and the gas flows in the center. By using Eq. (2.7) for the relative phase permeability, we obtain

$$f(p) = \frac{(1-g_s)\,p_s}{p_s - g_s p}\left[\sqrt{\tilde{\mu}^2 + \frac{\tilde{\mu}\,(p_s - p)}{\chi p}} + (1 - \tilde{\mu})\right]^2. \tag{3.17}$$

We consider (2.8) in the framework of one-dimensional flow and find

$$f(p) = \tilde{\mu}_* \frac{(1-g_s)\,p_s}{p_s - g_s p}\left(1 + \frac{p_s - p}{\chi p}\right). \tag{3.18}$$

Here the expressions for the dimensionless flows Q_{ie} have a form which coincides with (3.14) and (3.15) with the corresponding substitution of the pressure function into Eqs. (3.17) and (3.18).

Figure 1 shows calculated results for the dimensionless flow rate Q_{1e} as a function of the pressure drop Δp ($\Delta p = p_0 - p_e$) relative to experimental data [1]. The following values were used for the mixture properties and the experimental conditions: p_0 = 10 MPa, p_s = 3.8 MPa, ρ_1^0 = 700 kg/m³, ρ_{2s}^0 = 68 kg/m³, g_s = 0.11, $\mu_1 = 3.6\cdot10^{-4}$ Pa·sec, and $\mu_2 = 1.7\cdot10^{-5}$ Pa·sec [$\tilde{\mu}$ = 0.05 and χ = 0.88). The solid curve corresponds to the "gas bearing" model, and the dashed and the dot-dash curves to the inverted and equal-velocity models for flow of a gasified liquid in a porous medium. From Fig. 1 it can be seen that using the "gas bearing" model gives the better qualitative and quantitative description of the experimental data in the section where the flow rate rises rapidly and where the exit pressure from the porous medium becomes less than p_s = 3.8 MPa. We note that, according to the inverted and equal-velocity models, gas evolution leads to a reduced liquid flow rate (blocked flow). However, the experimental data on the flow rate reduction when the exit pressure is further reduced ($\Delta p \gtrsim$ 7.7 MPa and $p_e \lesssim$ 2.3 MPa) cannot be explained and are described by none of the models. This situation evidently is explained by the fact that the gas saturation is large under these conditions in the filtration-flow region next to the exit from the porous medium ($x = x_e$). Therefore gas flow continuity is lost in this region. The liquid condenses in to disconnected droplets. Pushing the disconnected droplets through the porous medium requires a pressure gradient estimated by

$$|\nabla p| \simeq \sigma/d^2.$$

For this case ($\sigma = 17.4\cdot10^{-3}$ kg/sec² and $d = 5\cdot10^{-5}$ m), we thus have

$$|\nabla p| \simeq 8\,\text{MPa/m}.$$

Figures 2 and 3 show the distributions of the pressure and the gas saturation in the porous medium in the two-dimensional plane case (ν = 1) for various pressures at the exit from the porous medium (curves 0-4 correspond to p_e = 3.8, 3.3, 3.0, 2.7, and 2.3 MPa). Values mentioned above were used for the other phase properties. The equations used in the two-phase region were those obtained from the "gas bearing" model. It can be seen that gas evolution greatly reduces the slope of the pressure-distribution curve, which in turn increases the flow rate, shown in Fig. 1, in the gas-evolution section ($\Delta p \gtrsim$ 6.2 MPa).

We note that the filtration of gasified liquid does not always occur under "superfluid" conditions, generally speaking, but only for a certain combination of material properties for the porous medium and the liquid.

4. Self-similar Solution. For two-dimensional plane flow ($\nu = 1$) we have from (1.5) and (1.6), with a consideration of (2.6) and the equation of state (1.4), that

$$\frac{\partial}{\partial t}\left[1 + S_2(\tilde{\rho}_{2s}^0 P - 1)\right] - \varkappa \frac{\partial}{\partial x}\left\{K\left[1 + S_2(K_v \tilde{\rho}_{2s}^0 P - 1)\right]\frac{\partial P}{\partial x}\right\} = 0; \tag{4.1}$$

$$\frac{\partial}{\partial t}\left[(1 - S_2)(1 - g)\right] - \varkappa \frac{\partial}{\partial x}\left[K(1 - S_2)(1 - g)\frac{\partial P}{\partial x}\right] = 0 \tag{4.2}$$

$$\left(g = g_s P,\ \tilde{\rho}_{2s}^0 = \frac{\rho_{2s}^0}{\rho_1^0},\ P = \frac{p}{p_s},\ \varkappa = \frac{k p_s}{m \mu_1},\ K = [\tilde{\mu} + (2 - \tilde{\mu}) S_2]/\tilde{\mu},\ K_v = \tilde{\mu} S_2 / K\right).$$

Now we examine the problem of filtering the gasified liquid in a semi-infinite region ($x \geqslant 0$) under the following initial and boundary conditions:

$$P(x, 0) = 1, \quad S_2(x, 0) = 0, \quad P(0, t) = P_e \quad (P_e < 1). \tag{4.3}$$

The initial condition in (4.3) for the pressure [$P(x, 0) = 1$ for $S_2(x, 0) = 0$] is required in this formulation for an incompressible fluid (ρ_1^0 = const). Actually if we set $P(x, 0) = P_e > 1$ and $P_e < 1$, then for some coordinate $x = x_s$, where x_s is the coordinate where gas evolution starts, $P(x_s, t)$ should equal unity. Because an incompressible liquid is filtering in the region $x_s < x < \infty$ [$S_2(x, t) = 0$] then the pressure should be uniform in this region [$P(x, t)$ = const]. Here the dimensionless pressure is equal to unity at $x = x_s$. Therefore, $P(x, t)$ should be unity in the whole region $x_s < x < \infty$.

This problem is self-similar. We introduce the self-similar variable $\xi = x/\sqrt{\varkappa t}$. The Eqs. (4.1) and (4.2) are written in the form

$$\frac{\xi}{2}\frac{d}{d\xi}\left[1 + S_2(\tilde{\rho}_{2s}^0 P - 1)\right] + \frac{d}{d\xi}\left\{K\left[1 + S_2(K_v \tilde{\rho}_{2s}^0 P - 1)\right]\frac{dP}{d\xi}\right\} = 0; \tag{4.4}$$

$$\frac{\xi}{2}\frac{d}{d\xi}\left[(1 - S_2)(1 - g_s P)\right] + \frac{d}{d\xi}\left[K(1 - S_2)(1 - g_s P)\frac{dP}{d\xi}\right] = 0 \tag{4.5}$$

with initial and boundary conditions

$$P(\infty) = 1, \quad S_2(\infty) = 0, \quad P(0) = P_e. \tag{4.6}$$

Flow rates through the boundary of the semi-infinite region, determined from the equations

$$u_{ie} = m S_{ie} v_{ie} = -\frac{k K_{ie}}{\mu_i}\left(\frac{\partial p}{\partial x}\right)_{x=0} \qquad (i = 1, 2),$$

under the filtration conditions in question are given by

$$u_{ie} = -\frac{k K_{ie} p_s}{\mu_i \sqrt{\varkappa t}}\left(\frac{dP}{d\xi}\right)_{\xi=0}. \tag{4.7}$$

From this it can be seen that, in order to calculate the flow rates u_{ie}, we must find the values of $dP/d\xi$ and S_2 for $\xi = 0$. This in turn requires obtaining the distribution of the pressure P and the gas saturation S_2 from the third-order nonlinear Eqs. (4.4) and (4.5) with boundary conditions (4.6). The system (4.4) and (4.5) was integrated numerically as follows. By linearizing and doing several transformations of this system, we obtain

$$(1 - g_s)\frac{d^2 P}{d\xi^2} - \frac{\xi}{2}\left[(1 - g_s)\frac{dS_2}{d\xi} + g_s \frac{dP}{d\xi}\right] = 0; \tag{4.8}$$

$$g_s \frac{dP}{d\xi} + \tilde{\rho}_{2s}^0 (1 - g_s)\frac{dS_2}{d\xi} = 0. \tag{4.9}$$

By considering the conditions (4.6), we obtain from (4.8) and (4.9) that

$$\frac{d^2 P}{d\xi^2} + \frac{\eta^2 \xi}{2}\frac{dP}{d\xi} = 0, \quad S_2 = \frac{g_s}{\tilde{\rho}_{2s}^0 (1 - g_s)}(1 - P) \quad \left(\eta^2 = \frac{g_s(1 - \tilde{\rho}_{2s}^0)}{(1 - g_s)\tilde{\rho}_{2s}^0}\right). \tag{4.10}$$

By solving the system (4.10) with the boundary conditions (4.6) at infinity, we find

$$P = 1 - A\int_{\xi}^{\infty}\exp(-\eta^2\xi^2/4)\,d\xi, \quad dP/d\xi = A\exp(-\eta^2\xi^2/4). \tag{4.11}$$

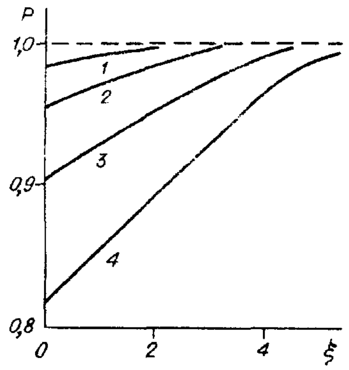

Fig. 4

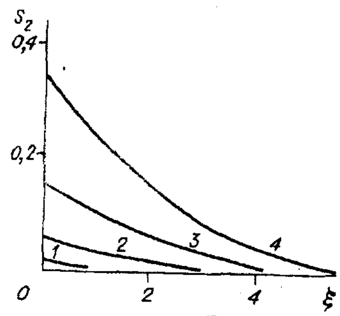

Fig. 5

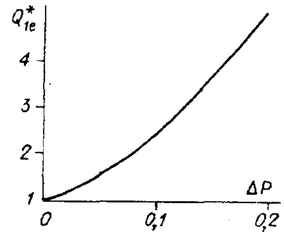

Fig. 6

We use these solutions to choose the initial Cauchy data for numerically integrating the system (4.4) and (4.5). Here the "initial" coordinate ξ_0 is chosen for a given value of A such that linearization conditions are fulfilled for $\xi > \xi_0$

$$(1-P),\ S_2/\tilde{\mu},\ \left|\frac{dS_2}{d\xi}\right|,\ \left|\frac{dP}{d\xi}\right|,\ \left|\frac{d^2P}{d\xi^2}\right| \ll 1. \tag{4.12}$$

Then the Cauchy problem is solved numerically for Eqs. (4.4) and (4.5) for the "initial" conditions ($\xi = \xi_0$)

$$P_0 = 1 - A\int_{\xi_0}^{\infty} \exp(-\eta^2\xi^2/4)\,d\xi,$$
$$\frac{dP}{d\xi} = A\exp(-\eta^2\xi_0^2/4),\quad S_{20} = (1-P_0)/\chi \tag{4.13}$$

in the range $0 \leqslant \xi \leqslant \xi_0$. By choosing various values of A, we obtain the distribution of the pressure and the gas separation for various pressure drops $\Delta P = 1 - P_e$.

Figures 4 and 5 show pressure and gas-saturation distributions as functions of the self-similarity coordinate for various pressure drops (curves 1-4 correspond to $\Delta P = 0.015$, 0.045, 0.095, and 0.18). We used values given in Paragraph 3 for the parameters of the gas–liquid system.

Based on solutions of (4.11), we can examine self-similar filtration conditions for a gasified liquid when the conditions of (4.12) are fulfilled over the whole flow region. It is not difficult to see that the condition

$$\Delta P \ll \tilde{\mu}\chi$$

must be fulfilled to realize this solution. Then the solution to (4.13) is written in the form

$$P = 1 - \frac{\eta\Delta P}{\sqrt{\pi}}\int_{\xi}^{\infty}\exp\left(-\frac{\eta^2\xi^2}{4}\right)d\xi,\qquad S_2 = \frac{1-P}{\chi}.$$

Here the flow rates through the boundary region are

$$u_{1e}^* = \frac{k\eta\Delta p}{\mu_1\sqrt{\pi\varkappa t}},\quad u_{2e}^* = \frac{S_{2e}^2 k\eta\Delta p}{\mu_2\sqrt{\pi\varkappa t}},\quad \Delta p = p_s - p_e,\quad S_{2e} = \Delta p/(\chi p_s). \tag{4.14}$$

We introduce a dimensionless parameter Q_{1e}^* for the liquid flow rate as a ratio of the flow rates determined from Eqs. (4.7) and (4.14):

$$Q_{1e}^* = \frac{u_{1e}}{u_{1e}^*} = \frac{K_{1e}}{\eta\Delta P}\left(\frac{dP}{d\xi}\right)_{\xi=0}.$$

The parameter Q_{1e}^* to some degree reflects the change of the flow rate due to the "gas bearing" effect under self-similar filtration conditions.

Figure 6 shows Q_{1e}^* as a function of the dimensionless pressure drop. Previous values were used for the properties of the gasified liquid and the porous medium. It can be seen that using the "gas bearing" model for self-similar filtration conditions leads to a significant growth in the liquid flow rate (Q_{1e}^* becomes greater than unity).

For comparison, we also introduce expressions for the flow rate of a "pure" liquid under elastic filtration conditions, which correspond to analogous initial and boundary conditions used to obtain (4.14):

$$u_e = \frac{k\Delta p}{\mu_1\sqrt{\pi \varkappa_c t}}, \quad \varkappa_c = \frac{k\rho_1^0 C_1^2}{m\mu_1}, \tag{4.15}$$

where C_1 is the sound speed in the liquid. Then the ratio of the liquid flow rates determined by (4.14) and (4.15) for the same Δp is

$$u_{1e}^*/u_e = C_1/C, \quad C = \sqrt{\frac{p_s(1-g_s)\,\tilde{\rho}_{2s}^0}{\rho_1^0 g_s(1-\tilde{\rho}_{2s}^0)}}. \tag{4.16}$$

Here C corresponds to the equilibrium sound speed for the gasified liquid near the start of gas evolution. In particular, for the mixture of n-hexane and carbon dioxide gas examined above, $C \approx 75$ m/sec ($C_1 \approx 10^3$ m/sec). That is, as follows from Eq. (4.16), the presence of gas evolution greatly increases the gas flow (by more than 10 times) compared to the case of filtration of a "pure" liquid with elastic conditions. This situation is related to the effect of the compressibility of the gas-saturated liquid due to gas evolution.

Thus, when a gas-saturated liquid is filtered through a porous medium and the exit pressure from the porous medium is reduced to values for the start of gas evolution, the sharp increase in the flow rate can be explained by the "gas bearing" effect, which leads to a "superfluid" filtering liquid. When the exit pressure is reduced further, the reduced flow rate is evidently related to a loss of fluid continuity (when the liquid condenses into separate droplets) in the porous medium near the exit.

The author thanks R. I. Nigmatulin for his attention to this work and for useful discussion of its results.

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